3.2

Sequences

A **sequence is a function** whose domain is the set of positive integers.

A sequence never ends. The numbers in the list are called the **terms** of the sequence.

__Recursive Formulas__: A way of defining a sequence is to assign a value to the first (or the first few) term(s) and specify the nth term by a formula or equation that involves one or more of the terms preceding it. Sequences defined this way are said to be defined **recursively** and the rule or formula is called a **recursive formula.**

Example: $s_1 = 1 \quad s_n = 4s_{n-1}$

$$S_2 = 4 \cdot S_{2-1}$$
$$S_2 = 4 \cdot S_1$$
$$S_2 = 4 \cdot 1 = 4$$

So, the sequence would be 1, 4, 16, 64, …

The Fibonacci sequence: 1, 1, 2, 3, 5,… can be defined recursively by:

$$S_1 = 1, \; S_2 = 1$$
$$S_n = S_{n-2} + S_{n-1} \quad \text{for } n > 2$$

Arithmetic Sequences

When the difference between successive terms of a sequence is always the same number, the sequence is called **arithmetic.** An **Arithmetic sequence** may be defined **recursively** as:

$a_1 = a$, $\quad a_n - a_{n-1} = d$ or as

$\boldsymbol{a_1 = a}$, $\quad \boldsymbol{a_n = a_{n-1} + d}$ where $a = a_1$ and d are real numbers. (**Recursive Formula**)

The number ***a* is the first term** and the number **d is called the common difference.**

For an arithmetic sequence $\{a_n\}$ whose first term is ***a*** and whose common difference is ***d***, the nth term is determined by the **explicit formula**…

$$a_n = a + (n - 1)d$$

Example:

For the following arithmetic sequence, find a) the common difference, b) the tenth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term.

1. -6, -2, 2, 6, 10, …

a) The difference between successive terms is 4.

b) $a_{10} = -6 + (10 - 1)(4) = 30$ (using explicit formula above)

c) The sequence is defined recursively by $a_1 = -6$ and $a_n = a_{n-1} + 4$ for all $n \geq 2$.

d) The sequence is defined explicitly by $a_n = -6 + (n - 1)(4) = 4n - 10$.

Geometric Sequences

When the ratio of successive terms of a sequence is always the same nonzero number, the sequence is called **geometric. A geometric sequence** may be defined **recursively** as

$$a_1 = a, \ \frac{a_n}{a_{n-1}} = r \qquad or \ \ as \qquad a_1 = a, \ \ a_n = r \bullet a_{n-1}$$

Where $a_1 = a$ and $r \neq 0$ are real numbers. The number ***a* is the first term,** and the nonzero number **r is called the common ratio**.

The terms of a geometric sequence with first term *a* and common ratio *r* follow the pattern:

$a, ar, ar^2, ar^3 \ldots$

For a geometric sequence $\{a_n\}$ whose first term is ***a*** and whose common ratio is ***r***, the nth term is determined by the **explicit formula**:

$$a_n = ar^{n-1}, \ \ r \neq 0$$

Example:

For the following geometric sequence find a) the common ratio, b) the tenth term, c) a recursive rule for the *n*th term, and d) an explicit rule for the *n*th term.

2. 3, 6, 12, 24, 48, …

a) The ratio between successive terms is 2.

b) $a_{10} = 3 \cdot 2^{10-1} = 3 \cdot 2^{9} = 1536$

c) The sequence is defined recursively by $a_1 = 3$ and $a_n = 2a_{n-1}$ for $n \geq 2$.

d) The sequence is defined explicitly by $a_n = 3 \cdot 2^{n-1}$.